Imaging Science Division Technical Memorandum

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Title: Optimum MTF of a Digital Camera Anti-Aliasing (Blur) Filter

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ABSTRACT:
This memo pulls together elements from several of my digital camera documents to attempt to answer the question of what is the optimum MTF of a digital camera anti-aliasing (or, blur) filter.

Keywords: digital camera, anti-aliasing filter, blur filter, MTF

KODAK MEMO

August 30, 1996

FROM: Jim Adams
TO: John Hamilton, Dave Kessler, Rodney Miller, Les Moore, Jim Mruk, Steve Noble, Ken Parulski, Joe Revelli, Steve Smith, Kevin Spaulding
SUBJECT: Optimum MTF of a Digital Camera Anti-Aliasing (Blur) Filter

This memo pulls together elements from several of my digital camera documents to attempt to answer the question of what is the optimum MTF of a digital camera anti-aliasing (or, blur) filter. While I state some conclusions for the first time in print in this document, I believe these statements are already existing in my previous works, if only between the lines. As such, I believe nothing very new is presented here.

This document has not received the same technical scrutiny from colleagues as my other works. Therefore, please feel free to discuss with me any points that do not seem clear or consistent. In terms of document control, I will probably bundle this memo with a few other ones I have written this year as a technical report near the end of the year. The version released in the technical report will incorporate any improvement made upon this work in the interim.

Bayer Pattern Sampling

In the Kodak Memo "Aliasing Analysis of Kodak Bayer Color Filter Array Pattern"[1], and Kodak Technical Reports "Edge Sharpening in Kodak Digital Cameras"[2] and "The Karnak CFA Interpolation Algorithm"[3], I develop a theory of signal sampling and recovery for the Bayer pattern along the lines presented by J. D. Gaskill in his book "Linear Systems, Fourier Transforms, and Optics"[4]. To summarize, consider the Bayer pattern superimposed on a Cartesian coordinate system in Fig. 1.

$$\mathrm{samp_G}(x,y) = \frac{1}{2}\mathrm{comb}\left(\frac{x+y}{2}, \frac{-x+y}{2}\right) \tag{4}$$

$$\mathrm{samp_B}(x,y) = \frac{1}{4}\mathrm{comb}\left(\frac{x}{2}, \frac{y-1}{2}\right) \tag{5}$$

The Fourier transforms of Eqs. 1, 4 and 5 are, respectively,

$$\mathrm{Samp_R}(\xi,\eta) = e^{-j2\pi\xi}\mathrm{comb}(2\xi, 2\eta) \tag{6}$$

$$\mathrm{Samp_G}(\xi,\eta) = \mathrm{comb}(\xi+\eta, -\xi+\eta) \tag{7}$$

$$\mathrm{Samp_B}(\xi,\eta) = e^{-j2\pi\eta}\mathrm{comb}(2\xi, 2\eta) \tag{8}$$

Plots of the absolute values of Eqs. 6, 7 and 8 will show the location, in frequency space, of the signal fundamental ($\xi = 0$, $\eta = 0$) and of all the potential harmonics. Figure 2 is a plot of $|\mathrm{Samp_R}(\xi,\eta)| = |\mathrm{Samp_B}(\xi,\eta)|$ and Fig. 3 is a plot of $|\mathrm{Samp_G}(\xi,\eta)|$.

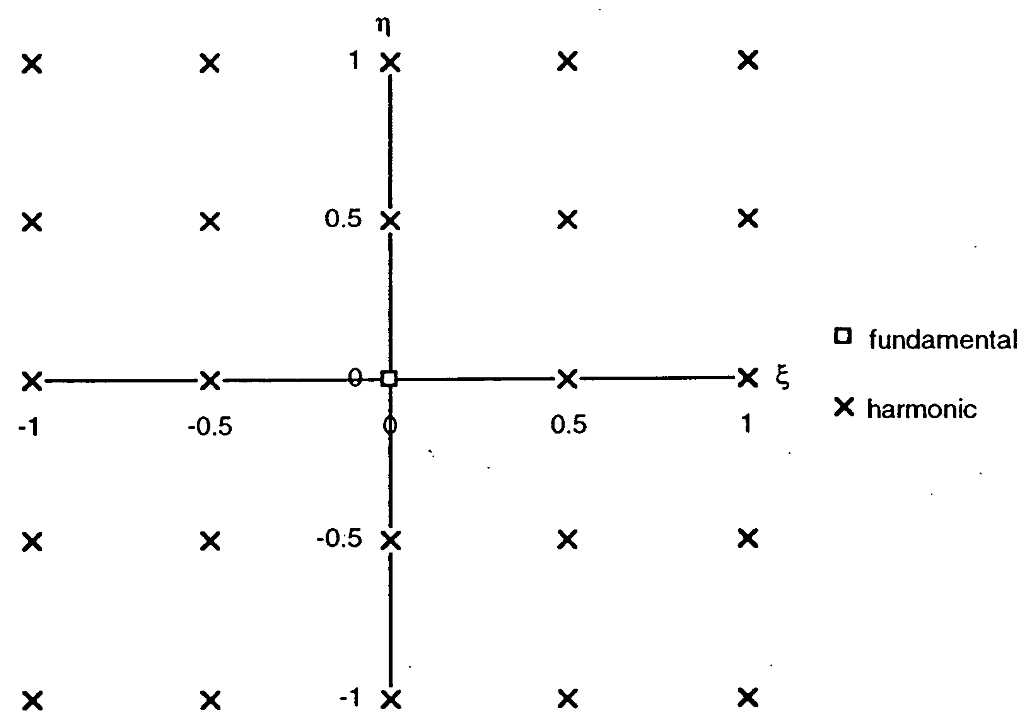

Figure 2. Plot of $|\mathrm{Samp_R}(\xi,\eta)| = |\mathrm{Samp_B}(\xi,\eta)|$. The axes are labeled in units of cycles/sample.

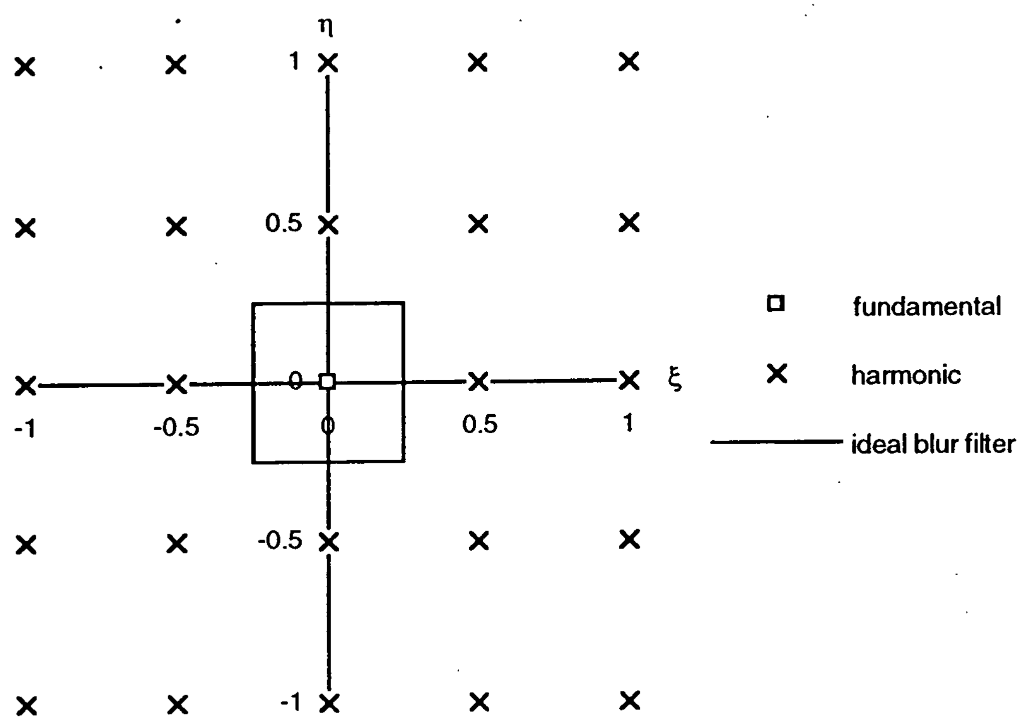

Figure 4. Plot of Ideal Anti-Aliasing Filter for Red and Blue Channels. The axes are labeled in units of cycles/sample.

among the three color channels. A simplifying assumption is that the vast majority of the luminance is contained in the green channel and that the remaining amounts in the red and blue channels can be ignored. Like all engineering approximations, this assumption is not entirely true and changes from application to application. However, it will serve well enough here. If we assume all the important luminance information is contained in the green channel, then we can describe a filter that blurs each channel equally and has a frequency response of Eq. 10. Such a filter will prevent aliasing due to Bayer pattern sampling.

Color Filter Array Interpolation

A tacit assumption made by the previous section is that all subsequent image processing is perfect with respect to spatial detail. Unfortunately, this is not the case. Color filter array (CFA) interpolation can produce significant artifacts in the reconstructed luminance of the image. Therefore, the design of the anti-aliasing filter must take this into account in order to avoid producing colored pixel artifacts as a result of CFA interpolation.

In the aforementioned "Edge Sharpening in Kodak Digital Cameras"[2] and "The Karnak CFA Interpolation Algorithm"[3] and in the Kodak Imaging Science Division Technical Memorandum "MTF of CFA Interpolation Kernels"[5], I derive analytical expressions for the modulation transfer function (MTF) of a number of CFA interpolation algorithms used in Kodak digital cameras. In general the MTF, Intp(ξ), of the entire class of CFA interpolation kernels used in Kodak Bayer pattern digital cameras can be described by Eq. 13.

$$\mathrm{Intp}(\xi) = \cos^2(\pi\xi) + k\sin^2(2\pi\xi) \tag{13}$$

The value k in Eq. 13 is different for each of the algorithms used. Table 1 summarizes appropriate values for k for a number of Kodak digital cameras.

Table 1. Values of k for Equation 13 for a Number of Kodak Digital Cameras.

k	Algorithm	Camera
0	Hibbard Laroche-Prescott	QuickTake 100 DCS-200 DCS-420
1/2	AH2 (Adams-Hamilton)	DC40 DC50
1/3	Karnak (Adams-Hamilton)	(none yet)

Figure 6 is a plot of Eq. 13 for the values of k listed in Table 1.

Table 2. Values of k_1, k_2 and k_3 for Equation 14 for a Number of Kodak CFA Interpolation Algorithms.

k_1	k_2	k_3	Algorithm
1	0	0	Hibbard
0	1/2	0	Laroche-Prescott
1/3	1/3	0	AH2 (Adams-Hamilton)
2/5	1/5	1/5	Karnak (Adams-Hamilton)

Figure 7 is a plot of Eq. 14 for the values of k_1, k_2 and k_3 listed in Table 2.

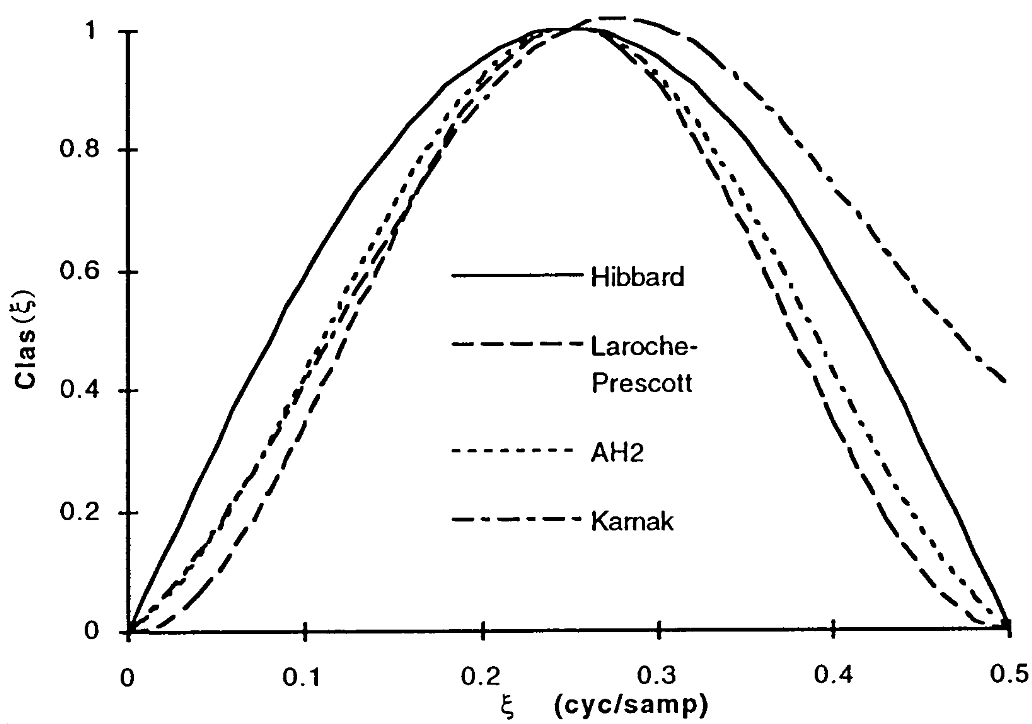

Figure 7. MTF of Various CFA Interpolation Classifiers Used in Kodak Digital Cameras

With the exception of the Karnak algorithm, it can be seen in Fig. 7 that there are two frequency ranges where the MTF drops below 20% modulation: near zero and near 0.5 cycles/sample. This means the classifier cannot tell whether a field of one pixel wide stripes (i.e., 0 cycles/sample in one direction and 0.5 cycles/sample in the orthogonal direction) is oriented horizontally or vertically. Also, at these frequencies the classifier value can be smaller than the level of noise in the image, resulting in random adaptive decisions. These effects are generally the sources of most colored pixel artifacts produced by the Hibbard, Laroche-Prescott and AH2 CFA interpolation algorithms. (There are advantages

$$\mathrm{tri}\left(\frac{x-x_0}{b}\right)=\begin{cases}0, & \left|\frac{x-x_0}{b}\right|\geq 1\\ 1-\left|\frac{x-x_0}{b}\right|, & \left|\frac{x-x_0}{b}\right|<1\end{cases} \tag{18}$$

A two dimensional version would be (compare to Eq. 10):

$$\mathrm{Blur}_{\mathrm{I}}(\xi,\eta)=\mathrm{trap}(\xi-\eta,\xi+\eta;0.25,0.4,0.25,0.4) \tag{19}$$

where

$$\mathrm{trap}(x-x_0,y-y_0;b_1,b_2,d_1,d_2)=\mathrm{trap}(x-x_0;b_1,b_2)\mathrm{trap}(y-y_0;d_1,d_2) \tag{20}$$

(A subtle point to note is that $\mathrm{Blur}_{\mathrm{I}}(\xi,0)$ of Eq. 19 is not equal to $\mathrm{Blur}_{\mathrm{I}}(\xi)$ of Eq. 15, but $\mathrm{Blur}_{\mathrm{I}}^2(\xi)$. This is not a significant difference in this discussion.)

Image simulation suggests that most important part of Eq. 15 is the region 0.4 cycles/sample and higher. Empirically, an approximation of Eq. 15, $\mathrm{Blur}_{\mathrm{S}}(\rho)$, that can be easily implemented in image simulation is given in Eq. 21.

$$\mathrm{Blur}_{\mathrm{S}}(\rho)=\mathrm{Gaus}\left(\frac{\rho}{0.44}\right) \tag{21}$$

where

$$\mathrm{Gaus}\left(\frac{x-x_0}{b}\right)=\exp\left[-\pi\left(\frac{x-x_0}{b}\right)^2\right] \tag{22}$$

and ρ is a frequency space radius (polar coordinates) in units of cycles/sample. In Photoshop, Eq. 21 is equivalent to a Gaussian blur with a radius of 0.8. Figure 8 is a plot of Eqs. 15 and 21.

The lower right image was generated using AH2 and Photoshop to apply a Gaussian blur with a radius of 0.8 pixels. The visibility of the remaining colored pixel artifacts is very low.

References

1. J. E. Adams, "Aliasing Analysis of Kodak Bayer Color Filter Array Pattern", Eastman Kodak Memo (1995).

2. J. E. Adams, "Edge Sharpening in Kodak Digital Cameras", Eastman Kodak Technical Report 300993V (1995).

3. J. E. Adams, "The Karnak CFA Interpolation Algorithm", Eastman Kodak Technical Report 301037R (1996).

4. Jack D. Gaskill, *Linear Systems, Fourier Transforms, and Optics,* John Wiley & Sons, New York, pp. 267–272 (1978).

5. J. E. Adams, "MTF of CFA Interpolation Kernels", Eastman Kodak Imaging Science Division Technical Memorandum 734-96008 (1996).

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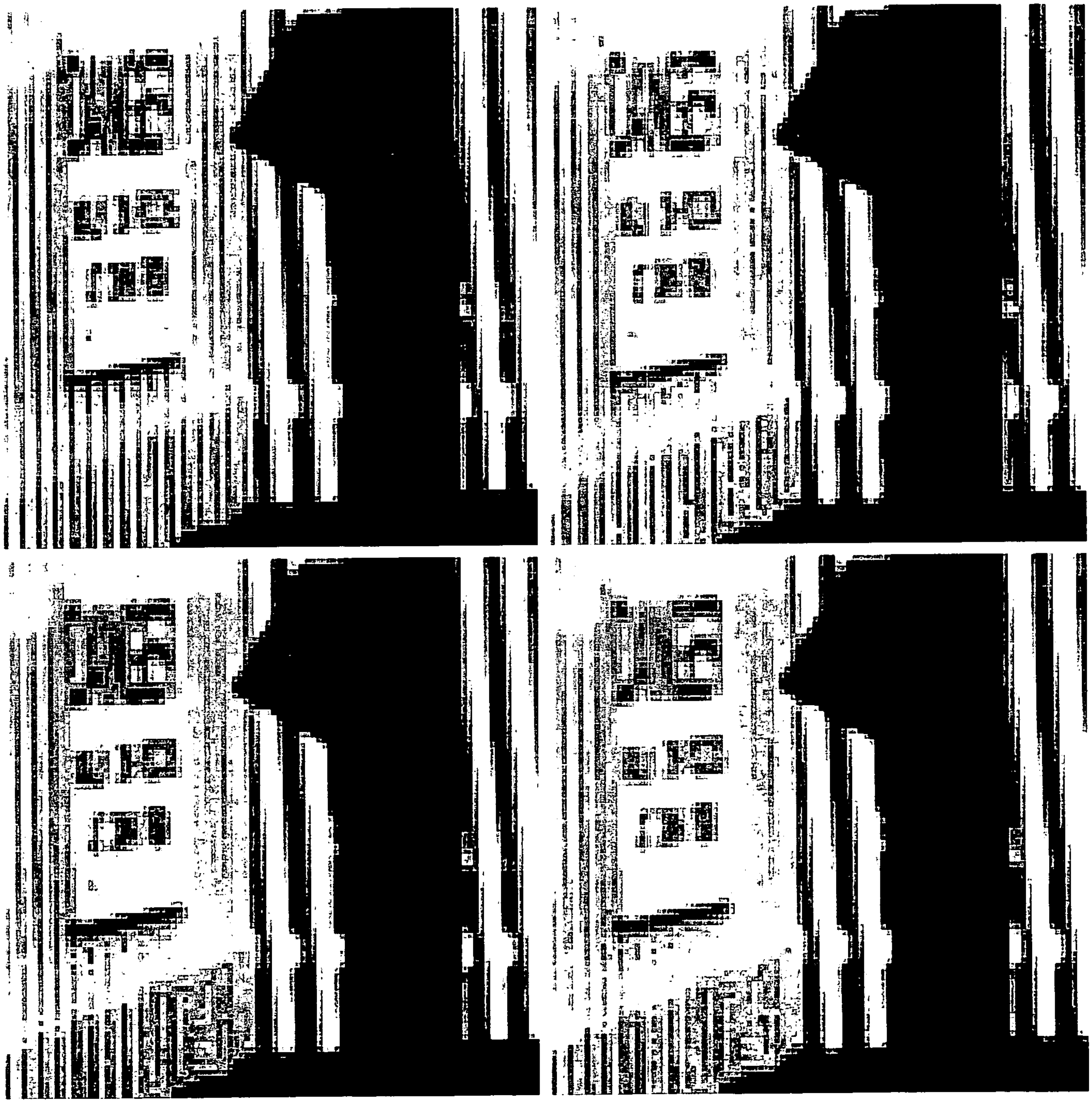

Figure 9. Upper Left: Original, Upper Right: DC50 Quartz Blur Filter, Lower Left: 1 2 1 Blur Filter, Lower Right: Gaus Blur Filter